

Complex mass definition and the concept of continuous mass

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Propagators of unstable particles are considered in the spectral representation which naturally follows from the concept of continuous mass. The spectral functions are found with the help of the most general formal and physical assumptions. Dressed propagators of unstable scalar, vector, and spinor fields are derived in an analytical way for a variant of parameter space. The structure of the propagators is in a correspondence with the complex mass scheme.

I. INTRODUCTION

Two standard definitions of the mass and width of unstable particles (UP), which are usually considered in the literature, have essentially different nature. The on-mass-shell (OMS) scheme defines the mass M and width Γ of a bosonic UP by the renormalization conditions [1]:

$$M^2 = M_0^2 + \Re\Pi(M^2), \quad M\Gamma = \frac{-\Im\Pi(M^2)}{1 - \Re\Pi'_s(M^2)}. \quad (1)$$

In the pole scheme (PS) the definitions of the mass and width are based on the complex-valued position of the propagator pole $s_R - M_0^2 - \Pi(s_R) = 0$. For instance, in the case of a vector UP a possible definition is as follows [1]:

$$s_R = M_\rho^2 - iM_\rho\Gamma_\rho, \text{ where } s_R = M_0^2 + \Pi(s_R). \quad (2)$$

It should be noted that this definition of the pole mass M_ρ and width Γ_ρ (also called m_2, Γ_2 or $\tilde{M}_Z, \tilde{\Gamma}_Z$) is not unique. There has been considerable discussion concerning definition of the vector-boson mass [2–8]. It was shown that OMS scheme contains spurious higher-order gauge-dependent terms while PS provides gauge invariant definition. Later on the PS definition was considered in detail [9–11] and gauge-invariant treatment was developed in the frame of the complex-mass scheme (CMS) [12, 13].

An alternative, semi-phenomenological, scheme based on the hypothesis of continuous (smeared) mass of UP was considered in [14, 15] (and references therein). In this approach, the physical values of the mass and width are related to the parameters of continuous mass distribution. In the model the propagators of UP have spectral-representation form, so that the standard problems transform to constructing the spectral function which describes the mass distribution. The effective theory of UP was developed on the base of this approach in [16], where the method of factorization of widths and cross-sections is presented.

In this work, we show that the concept of continuous mass along with some natural assumptions (which are considered in the next section) lead to the pole scheme with complex mass definition. This result follows directly from the propagators in spectral representation

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which, in turn, follows from the quantum-field realization of the mass-smearing concept [14, 15].

The paper is organized as follows. In section II we present the principal elements of the approach under consideration and analyze the general structure of the propagators in the model. The structure of the bosonic and fermionic propagators is considered in sections III and IV respectively. Some conclusions concerning the physical status of the results are made in section V.

II. GENERAL STRUCTURE OF THE PROPAGATOR OF AN UNSTABLE PARTICLE

Traditional way to construct the dressed propagator of UP is the Dyson summation, which introduces the width and redefines the mass of UP. This procedure runs into some problems widely discussed in the literature. Let us consider one of the formal sources of the difficulties which follow from this summation. In the simplest case of a scalar UP this procedure looks like

$$\begin{aligned} D_{(1)}(q) &= D_0(q) \sum_{k=0}^{\infty} (-i\Pi_{(1)}(q)D_0(q))^k \\ &= \frac{D_0(q)}{1 + i\Pi_{(1)}(q)D_0(q)} = \frac{i}{q^2 - M_0^2 - \Pi_{(1)}(q)}, \end{aligned} \quad (3)$$

where $D_0(q) = i(q^2 - M_0^2 + i\epsilon)^{-1}$ is the propagator of a scalar free field and $\Pi_{(1)}(q)$ is the one-particle-irreducible self-energy. The dressed propagator (3) is formally incorrect in the near-resonance range, because of the finite radius of convergence of the series

$$\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k = 1 + z + z^2 + \dots, \quad |z| < 1. \quad (4)$$

From Eqns. (3) and (4) it follows that the variable $z = \Pi_{(1)}(q)/(q^2 - M_0^2)$ should be correctly redefined before summation and one can not use the procedure in the region $z > 1$ after the redefinition. Thus, the dressed propagator in the form (3) contains an assumption of infinite series, that is, UP is described as a non-perturbative object. This, evidently, leads to difficulties in the scheme of sequential fixed-order calculations, which exhibit themselves in the violation of gauge invariance. As was pointed out in the previous section, such problems have been under considerable discussion for many years.

An alternative approach is based on the spectral representation of the propagator of UP. It has a long history [17–21] and treats UP as a non-perturbative state or effective field (asymptotic free field [20, 21]). Here, we consider this approach in the framework of the model of UP with continuous (smeared) mass [14, 15]. In the works [14, 15], the effective field function of UP is a continuous superposition of ordinary ones with a universal weight function $\omega(m)$ which describes the distribution of the mass parameter m . The value m was interpreted as a random mass with continuous distribution from some threshold to infinity (physical range definition). Here, we consider some modification of this approach which leads to significant changes in the effective theory of UP. We consider a non-universal weight function $\omega(q, m)$, that is the function which depends on the momentum q of the state.

The second modification is the extension of the allowed values of the mass parameter m^2 from $[m_0^2, +\infty)$ to $(-\infty, +\infty)$. So, the parameter m^2 loses its previous simple interpretation. It acquires, however, the status of the momentum squared q^2 in the whole Minkowski space. Further we show that this modifications lead to PS definition.

The field function of UP in the momentum representation is defined in exact analogy with the definition in [14] except for the above mentioned modification:

$$\phi(q) = \int_{-\infty}^{+\infty} \phi(q, m^2) \omega(q, m^2) dm^2. \quad (5)$$

The canonical commutation relations are not modified—they contain, as in Ref. [14], an additional delta-function $\delta(m^2 - m'^2)$. Using the definition (5) and the commutation relations from [14, 15], one get the propagator of a scalar unstable field in spectral representation. Starting from the standard definition of the Green's function, by straightforward calculations we get the Lehmann-like spectral representation of the propagator [15, 16]:

$$D(q) = i \int dx \exp(-iqx) \langle 0 | \hat{T} \phi(x) \phi(0) | 0 \rangle = i \int_{-\infty}^{+\infty} \frac{\rho(q, m^2) dm^2}{q^2 - m^2 + i\epsilon}, \quad (6)$$

where $\rho(q, m^2) = |\omega(q, m^2)|^2$. For the case $m^2 < 0$, that is $m = i|m|$, we leave the physical range of the continuous (smearing) mass. It should be noted, that the expression (6) is applicable for any values of q^2 and m^2 without restrictions. Moreover, it is not directly connected with the expansion of the Dyson type and perturbative constructions. So, the correct definition of the function $\rho(q, m^2)$ makes it possible to escape some problems which arise in a traditional approach.

Now, let us consider possible physical consequences of the presence of negative mass parameter $m^2 < 0$ in the spectral representation (5) and (6). Negative component of the spectrum leads to the states with imaginary mass parameters which usually interpreted as tachyon states. The problem of the existence of tachyons is under considerable discussion in the last decades. The main attention is paid to the principal problems, such as a violation of causality, tachyon vacuum, and radiation instability. There is no unique and consistent quantum field theory of tachyons, and various approaches are suggested to overcome the above mentioned difficulties. We note, that the first problem relates to UP as an observable object. Further we show that such object in the framework of our effective model is described by the positive mass square $M^2(q)$ and width $\Gamma(q)$. However, quantum field description of UP as the continuous superposition with a tachyon component encounters the problem of instability. In the third section, we estimate the tachyon fraction for the case of fundamental UP. It is found to be rather small; for instance, in the case of Z boson it is approximately a percent. The analysis of an expression for the tachyon fraction leads to an interesting conclusion: tachyon instability is intrinsic property of UP; it can be interpreted as the cause of unstable particle decay.

The principal problem of the approach under consideration is to define the spectral function $\rho(q, m^2)$ which is the main characteristic of UP. Some phenomenological definitions were considered in [14]. Here, we consider the construction of this function with an account of the most general formal and physical arguments. First of all, we choose two-parametric distributions, where the parameters are directly connected with the mean values of mass and width of UP. These parameters are some functions of the four-momentum, that is $M(q)$ and $\Gamma(q)$ are q -dependent characteristics of UP. The probability density $\rho(q, m^2)$ has to be

normalized for any q . If $q \rightarrow q_0$, where q_0 is the threshold value of the momentum, the value $\Gamma(q) \rightarrow \Gamma(q_0) = 0$. So, the particle become stable, its mass go to a fixed value $M(q_0) = M_0$ and the function $\rho(q, m^2)$ behaves as a delta-function $\delta(m - M_0)$. There are three well-known approximations of the delta-function which are continuously differentiable and normalized. These are as follows [22]:

$$(a) \frac{\alpha}{\pi(1 + \alpha^2 x^2)}; \quad (b) \frac{\alpha}{\sqrt{\pi}} \exp(-\alpha^2 x^2); \quad (c) \frac{\alpha \sin \alpha x}{\pi \alpha x} \quad (\alpha \rightarrow \infty). \quad (7)$$

It is difficult to find the physical interpretation for the case of the oscillating distribution (c) in Eq. (7). As was discussed in [14], the exponential distribution (b) can be motivated when the smearing of mass is caused by stochastic mechanism of UP interaction with vacuum. It was noted that fast decreasing of the exponent does not describe some deeply virtual processes. One can see that the distribution (a) is the well-known Lorentzian distribution, where $\alpha \sim (\Gamma(q))^{-1}$ and $x = m^2 - M^2(q)$ or $x = m - M(q)$. It is easy to check that this function is normalized to unity. So, the function (a) in Eq. (7) satisfies above mentioned formal and physical requirements and will be used in derivation of the propagators of UP.

III. PROPAGATORS OF BOSONIC UNSTABLE PARTICLES

In this section, we consider the structure of the propagators of scalar and vector UP. From Eq. (1) and dimension requirement we have to choose the parametrization $x = m^2 - M^2(q)$ and $\alpha = (q\Gamma(q))^{-1}$ or $\alpha = (\Gamma(q))^{-2}$. Then, the probability density (7a) takes the form

$$\rho(q, m^2) = \frac{1}{\pi} \frac{\alpha^{-1}}{x^2 + \alpha^{-2}} = \frac{1}{\pi} \frac{q\Gamma(q)}{[m^2 - M^2(q)]^2 + q^2\Gamma^2(q)}, \quad (8)$$

where $M(q)$ and $\Gamma(q)$ are q -dependent parameters of the distribution. According to Eqs. (6) and (8) the propagator of a scalar UP can be written as

$$D(q) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{P(q) dm^2}{(q^2 - m^2 + i\epsilon)\{[m^2 - M^2(q)]^2 + P^2(q)\}}, \quad (9)$$

where $P(q) = q\Gamma(q)$ and we omit i for the simplicity of consideration. The integral in (9) can be calculated with the help of the integration rule

$$\int_{-\infty}^{+\infty} \frac{f(x) dx}{x \pm i\epsilon} = \mp i\pi f(0) + \mathcal{P} \int_{-\infty}^{+\infty} \frac{f(x)}{x} dx, \quad (10)$$

which follows from the Sokhotski-Plemelj formula. In Eq. (10) $\mathcal{P} \int$ stand for the principal part of the integral. As a result we have:

$$\begin{aligned} \Im D(q) &= -\pi \rho(q, q^2) = \frac{-P(q)}{[q^2 - M^2(q)]^2 + P^2(q)}; \\ \Re D(q) &= \mathcal{P} \int_{-\infty}^{+\infty} \frac{\rho(q, m^2) dm^2}{q^2 - m^2} = \frac{q^2 - M^2(q)}{[q^2 - M^2(q)]^2 + P^2(q)}. \end{aligned} \quad (11)$$

It is easy to check that Eqs. (11) result in the following expression for the dressed propagator of a scalar UP:

$$D(q) = \frac{1}{q^2 - M^2(q) + iP(q)}, \quad (12)$$

where $P(q) = q\Gamma(q)$. The same result can be got in a more simple way with the help of contour integration. Let us rewrite the function $D(q)$ as follows:

$$D(q) = - \int_{-\infty}^{+\infty} \frac{\rho(q, m^2) dm^2}{m^2 - (q^2 + i\epsilon)} = -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{P(q) dm^2}{(m^2 - z_0)(m^2 - z_+)(m^2 - z_-)}, \quad (13)$$

where $z_0 = q^2 + i\epsilon$ and $z_{\pm} = M^2(q) \pm iP(q)$. Analytical continuation of the integrand function $f(z)$, where $m^2 \rightarrow z$, has three poles z_0, z_+, z_- in the complex plane. It decreases as $1/z^2$ for $z \rightarrow \infty$, that is, it satisfies the condition $|f(z)| < M/|z|^{1+\delta}$ for $|z| > R_0$, where M and δ are positive numbers and $R_0 \rightarrow \infty$. So, we can rearrange $D(q)$ as follows:

$$D(q) = \mp \frac{P(q)}{\pi} \oint_{C_{\pm}} \frac{dz}{(z - z_0)(z - z_+)(z - z_-)} = 2\pi i \sum_{k=z_0, z_+, z_-} \text{Res}(f(z), z_k), \quad (14)$$

where $\text{Res}(f(z), z_k)$ is the residue at the pole z_k and C_{\pm} is a contour in the upper (C_+) or lower (C_-) half of the complex z -plane. The simplest way to perform the integration is to go along the contour C_- which contains only one pole z_- :

$$\begin{aligned} D(q) &= \frac{P(q)}{\pi} \oint_{C_-} \frac{dz}{(z - z_-)(z - z_+)(z - z_0)} \frac{1}{(z - z_+)(z - z_0)} \\ &= \frac{2iP(q)}{(z_- - z_+)(z_- - z_0)} = \frac{1}{q^2 - M^2(q) + iP(q)}. \end{aligned} \quad (15)$$

In Eqs. (15) we have used the equality $z_- - z_+ = -2iP(q)$. One can check that the same result follows from the integration along the contour C_+ .

Thus, UP can be described at two different hierarchical levels—“fundamental” level by the spectral representation (9) and phenomenological one by the effective theory after integrating out unobservable mass parameter m^2 . In the framework of this theory, UP is described by the observed physical values $M^2(q)$ and $\Gamma(q)$, which can always be defined as a positive quantity. So, at this phenomenological level UP has no explicit tachyonic content that could lead to the problems noted in the Introduction. At the first level, however, the approach under consideration deals with explicitly tachyonic component in the field function of UP. As it was noted earlier, it can lead to an instability of the system. Let us estimate the tachyonic fraction, that is the probability of the states with $m^2 < 0$. From the Eq.(8) it follows that this probability $P_t(M(q), \Gamma(q))$ under the condition $M(q)/\Gamma(q) \ll 1$ is as follows:

$$P_t(M(q), \Gamma(q)) = \int_{-\infty}^0 \rho(m^2; M^2(q), \Gamma(q)) dm^2 \approx \frac{\Gamma(q)}{\pi M(q)}. \quad (16)$$

Thus, for the majority of fundamental particles this value is very small. However, for the case of vector boson (Z and W) and t -quark the tachyon fraction is appreciable (10^{-2}). From the Eq.(16) it follows that this fraction is defined by a value of $\Gamma(q)/M(q)$, that is the value which quantifies the effects of instability or finite-width effects (FWE) in the processes with UP participation. This makes it possible to suggest the direct connection between the tachyon component and instability of the particle. The decay of UP can be caused, for instance, by the instability of the tachyon vacuum.

To define the structure of the vector propagator, we assume that the spectral function $\rho(q, m^2)$ is the same as for a scalar UP. Using the standard vector propagator for a free

vector particle with a fixed mass, we get:

$$D_{\mu\nu}(q) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{-g_{\mu\nu} + q_\mu q_\nu / (m^2 - i\epsilon)}{q^2 - m^2 + i\epsilon} \frac{P(q) dm^2}{[m^2 - M^2(q)]^2 + P^2(q)}. \quad (17)$$

In the propagator term $q_\mu q_\nu / (m^2 - i\epsilon)$ we use the same rule of going around pole as in the denominator $q^2 - (m^2 - i\epsilon)$. The integral in Eq. (17) can be evaluated with the help of the formula (10), however, it is easier to do it using the method of contour integration. The integration along the lower contour C_- gives:

$$\begin{aligned} D_{\mu\nu}(q) &= -\frac{P(q)}{\pi} \oint_{C_-} \frac{(g_{\mu\nu} - q_\mu q_\nu / (z - i\epsilon)) dz}{(z - z_-)(z - z_+)(z - z_0)} \\ &= -2iP(q) \frac{g_{\mu\nu} - q_\mu q_\nu / (z_-)}{(z_- - z_+)(z_- - z_0)} = \frac{-g_{\mu\nu} + q_\mu q_\nu / (M^2(q) - iP(q))}{q^2 - M^2(q) + iP(q)}. \end{aligned} \quad (18)$$

One can check that the integration along the upper contour C_+ or with the help of the formula (10) leads to the same result.

We can see that both the scalar and vector propagators of UP can be represented in the form with universal complex mass squared:

$$D(q) = \frac{1}{q^2 - M_P^2(q)}; \quad D_{\mu\nu}(q) = \frac{-g_{\mu\nu} + q_\mu q_\nu / M_P^2(q)}{q^2 - M_P^2(q)}, \quad (19)$$

where $M_P^2(q) = M^2(q) - iP(q)$. Note that the dressed propagator of a bosonic UP can be get from the “free” propagator by the substitution $m^2 - i\epsilon \rightarrow M^2(q) - iP(q)$. The inverse substitution, $P(q) \rightarrow i0$, $M^2(q) \rightarrow M_0^2$, leads to the “free” propagator, if $q \rightarrow q_0$.

The q -dependent parameters $M(q)$ and $\Gamma(q)$ of the propagator pole $s_R(q) = M_P^2(q) = M^2(q) - iq\Gamma(q)$ are a generalization of the traditional pole definition of the mass and width of UP, which is based on the complex-valued position of the propagator pole $s_R - M_0^2 - \Pi(s_R) = 0$ [1]. It should be noted that such pole mass and width are defined in the non-physical region of four-momentum squared $q^2 = s_R$. In our analysis, propagator pole is q -dependent, $s_R(q) = M^2(q) - iq\Gamma(q)$, so we have q -dependent (running) pole definition of the mass and width. At fixed point $q^2 = M_\rho^2$ we get an ordinary definition $s_R(M_\rho) = M^2(M_\rho) - M_\rho\Gamma(M_\rho)$. The correctness and gauge invariance of such definition are strongly stipulated by the definition of the functions $M(q)$ and $\Gamma(q)$.

The traditional pole definition of the mass and width follows from the poles in the S -matrix. In practical calculations we deal with an expansion of the S -matrix, which is defined by the interaction Lagrangian. For instance, the process $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ is described at the tree level by the second order term and the pole in the S -matrix in this approximation is defined by the pole of the boson propagator. So, this pole depends on the scheme of inclusion of the mass and width terms into the dressed propagator of Z -boson (see the discussion in Refs. [23, 24]). We consider q^2 dependent mass and width as the most general case. But, this scheme is not obligatory for the consideration and we can include fixed mass and width terms which is valid in the vicinity of the resonance. As a result, we get a unique pole of the S -matrix and a pole definition of the fixed mass and width.

IV. PROPAGATOR OF A SPINOR UNSTABLE PARTICLE

The propagator of a free fermion can be represented in two equivalent forms:

$$\hat{D}(q) = \frac{1}{\hat{q} - m + i\epsilon} = \frac{\hat{q} + m - i\epsilon}{q^2 - (m - i\epsilon)^2}. \quad (20)$$

According to the heuristic rule for constructing the dressed propagator, we have to make the substitution $m \rightarrow M(q)$ and $i\epsilon \rightarrow i\Gamma(q)$, where the dimension $[\epsilon] = [m]$ was taken into account. Then, the dressed propagator of the spinor UP takes the form

$$\hat{D}(q) = \frac{\hat{q} + M(q) - i\Gamma(q)}{q^2 - (M(q) - i\Gamma(q))^2} = \frac{\hat{q} + M_P(q)}{q^2 - M_P^2(q)}, \quad (21)$$

where $M_P(q) = M(q) - i\Gamma(q)$ is q -dependent pole-type complex mass. Now, we show that the expression (21) can be derived in a more systematic way with the help of the spectral representation:

$$\hat{D}(q) = \int \frac{\hat{q} + m}{q^2 - (m - i\epsilon)^2} \rho(q, m) dm, \quad (22)$$

where the integration range is not defined yet. The spectral function $\rho(q, m)$ for fermions differs from the bosonic one, because of another parametrization $M(q) = M_0 + \Re\Sigma(q)$ and $\Gamma(q) = \Im\Sigma(q)$. So, we have to take $x = m - M(q)$ and $\alpha = \Gamma(q)$ in the general expression for $\rho(q, m)$ in Eq. (7a). As a result, the spectral function for the case of the spinor UP is as follows:

$$\rho(q, m) = \frac{1}{\pi} \frac{\Gamma(q)}{[m - M(q)]^2 + \Gamma^2(q)} = \frac{1}{\pi} \frac{\Gamma(q)}{(m - M_-(q))(m - M_+(q))}, \quad (23)$$

where $M_{\pm}(q) = M(q) \pm \Gamma(q)$. The main difference between boson and spinor cases is a presence of the linear term m instead the quadratic one m^2 , which is defined at the whole real axis $m^2 \in (-\infty, +\infty)$. Here, we restrict the problem by the formal definition and consider a straightforward relation between the bosonic parameter range and spinor one. Thus, we have two intervals $(\pm i\infty, i0; 0, \infty)$ for the value m . In the method of contour integration the signs \pm correspond to integration along the contours C_{\pm} , which enclose the first or fourth quadrants of the complex plane. Then, from Eqs. (22) and (23) it follows:

$$\hat{D}_{\pm}(q) = \pm \frac{\Gamma(q)}{\pi} \int_{C_{\pm}} \frac{(\hat{q} + z) dz}{(z^2 - z_0^2)(z - z_-)(z - z_+)}, \quad (24)$$

where $z_0^2 = q^2 + i\epsilon$, $z_{\pm} = M_{\pm}(q)$ and C_{\pm} are the above described contours. By simple and straightforward calculations one can see that the result (21) follows from the integration along the contour C_- , while the integration along the C_+ leads to non-physical result. This is likely caused by the presence of the branch point z_0^2 in the first quadrant. From Eq. (24) it follows:

$$\begin{aligned} \hat{D}_-(q) &= -\frac{\Gamma(q)}{\pi} \int_{C_-} \frac{dz}{z - z_-} \frac{\hat{q} + z}{(z^2 - z_0^2)(z - z_+)} \\ &= -2i\Gamma(q) \frac{\hat{q} + z_-}{(z_-^2 - z_0^2)(z_- - z_+)} = \frac{\hat{q} + M_P(q)}{q^2 - M_P^2(q)}. \end{aligned} \quad (25)$$

The integration along the contour C_+ leads to a more complicated expression. Note, that while the spinor q -dependent complex mass $M_P(q) = M(q) - i\Gamma(q)$ differs from the bosonic one, it has, however, the same pole-type structure. Then, the pole definition of the mass and width of the spinor UP is $M_P(M_\rho) = M(M_\rho) - i\Gamma(M_\rho)$.

V. CONCLUSION

The definitions of the mass and width of UP, as a rule, are closely connected with the construction of the dressed propagators. There are two main definitions, which follows from the on-shell re-normalization and pole structure of the propagators (see Introduction). We considered the general structure of the propagators of UP in the phenomenological approach based on the spectral representation. The spectral function was defined with account of the some formal and physical suggestions which naturally arise in the model of UP with continuous mass.

In this work, we analyzed a specific case—the spectral function depends on q -dependent parameters and random mass variable m^2 defined in the interval $(-\infty, +\infty)$. So, the variable m can be imaginary, which is beyond the physical domain in the model of UP with continuous (smeared) mass [14]. It was shown that this leads to the q -dependent complex mass which gives the pole definition of the mass and width of UP. We also suggest that the imaginary component of mass spectrum, which corresponds to tachyonic states, causes the instability of particle.

We have proved a simple heuristic rule for constructing the dressed propagators, which is based on the definition of q -dependent mass and width of UP. Physical q -dependence of the UP width makes it possible to introduce naturally the stable particle limit near threshold momentum and the narrow width approximation $\Gamma(q^2) \rightarrow 0$ when $q^2 \rightarrow q_0^2$. Note, however, that the physical nature of the random mass parameter m^2 is vague at $m^2 < 0$.

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